Calculus I **Lab Assignment**

For this lab you will use the free graphing software Gnuplot, Octave—a free clone of the commercial numerical computing software MATLAB® (with which Octave is syntax-compatible)—and Maxima, a free computer algebra system (CAS). You can get the latest versions of those applications from their homepages:

- Gnuplot: `http://gnuplot.info/`
- Octave: `https://www.gnu.org/software/octave/`
- Maxima: `https://maxima.sourceforge.io/`

Download the appropriate versions for your operating system and follow the installation instructions. If you are using Windows then make sure when installing Octave that the **Octave Forge** component is selected for installation (if that option is available in the **Choose Components** dialog window).

1. Open Gnuplot, and at the `gnuplot>` prompt enter the following commands:

```
set zeroaxis
set xlabel 'x'
set ylabel 'y'
set samples 5000
```

If nothing happens then that means you entered everything correctly. The above commands draw the x-axis, label the x-axis and the y-axis, and tell Gnuplot how many points to select for plotting over a given interval, respectively.

The usual way to create graphs in Gnuplot is with the `plot` command:

`plot <range> <comma-separated list of functions>`

For a function $y = f(x)$, <*range*> is the range of x values (and optionally the range of y values) over which to plot. To specify an x range, use an expression of the form $[a:b]$, for some numbers $a < b$. This will cause the graph to be plotted for $a \le x \le b$. To specify an x range and a y range, use an expression of the form $[a:b][c:d]$, for some numbers $a < b$ and $c < d$. This will cause the graph to be plotted for $a \le x \le b$ and $c \le y \le d$.

Function definitions use the x variable in combination with mathematical operators, some of which are listed below:

Symbol/Command	**Operation**	**Example**	**Result**
`+`	Addition	`2+3`	5
`-`	Subtraction	`3-2`	1
`*`	Multiplication	`2*3`	6
`/`	Division	`4/2`	2
`**`	Power	`2**3`	$2^3 = 8$
`sin(x)`	$\sin x$	`sin(2*x)`	$\sin 2x$
`cos(x)`	$\cos x$	`cos(x**2)`	$\cos x^2$
`tan(x)`	$\tan x$	`tan(pi*x)`	$\tan \pi x$
`asin(x)`	$\sin^{-1} x$	`asin(2*x)`	$\sin^{-1} 2x$
`acos(x)`	$\cos^{-1} x$	`acos(x**2)`	$\cos^{-1} x^2$
`atan(x)`	$\tan^{-1} x$	`atan(pi*x)`	$\tan^{-1} \pi x$
`exp(x)`	e^x	`exp(2)`	e^2
`log(x)`	$\ln x$	`log(pi)`	$\ln \pi$
`sqrt(x)`	$\sqrt{x}$	`sqrt(1+x**2)`	$\sqrt{1+x^2}$

Note that the special keyword "pi" denotes the value of π. The trigonometric functions use radians.

Note also that Gnuplot has no built-in functions for the reciprocal trigonometric functions (sec, csc, cot). So, for example, to plot $\sec x$ you would have to use $1/\cos(x)$.

2. Plot the function $y = x^3 - 3x^2 - 5x + 1$ over the interval $[-3, 5]$ by entering this command at the `gnuplot>` prompt:

```
plot [-3:5] x**3 - 3*x**2 - 5*x + 1
```

Print this graph by selecting **Print** near the top of the main Gnuplot window and enter **png** in the **Terminal type?** textfield, then hit OK to get the Print Setup dialog and print to the default printer.

3. Plot the given functions over the indicated interval, then print out the plots:

(a) $y = xe^{-x}$ over $[0, 5]$

(b) $y = 6\cos 6x + \sin 4x$ over $[0, 2\pi]$

(c) $y = \dfrac{\ln x}{\sqrt{x}}$ over $[1, 100]$

(d) $y = \dfrac{1}{x}$ over $[-10, 10]$, with y restricted to $[-10, 10]$

4. Plot $y = e^{-x^2}$ and $y = \frac{1}{1+x^2}$ together on the same plot, over the interval $[-4, 4]$. Print out the plot.

5. Quit Gnuplot by typing `quit` at the `gnuplot>` prompt.

6. Open Octave, which presents you with a command window. Octave and Maxima use mostly the same syntax for operations and functions. One difference is that Octave uses the special notation `pi` for the number π (whereas Maxima uses `%pi`). Commands in Octave are submitted by the Enter key. In Octave you can use the up and down arrow keys to navigate through your command history.

Octave and Maxima use the same function syntax as Gnuplot, but they do have built-in functions for the reciprocal trigonometric functions. Octave and Maxima also use the same operator syntax as Gnuplot with the exception of exponentiation:

Operation/Function	Symbol/Command	Normal math	Octave syntax	Maxima syntax
Exponentiation	`^`	x^3	`x^3`	`x^3`
$\sec x$	`sec(x)`	$\sec \pi x$	`sec(pi*x)`	`sec(%pi*x)`
$\csc x$	`csc(x)`	$\csc 4x^2$	`csc(4*x^2)`	`csc(4*x^2)`
$\cot x$	`cot(x)`	$\cot^2 x$	`cot(x)^2`	`cot(x)^2`

7. Enable long decimal support and disable paging of results with these commands in the Octave window:

```
format long
more off
```

Then evaluate $\frac{1}{\sqrt{2\pi}} e^{-\pi^2/2}$ with this command:

```
exp((-pi^2)/2)/sqrt(2*pi)
```

You should see output like this:

```
ans =  0.00286914634635448
```

8. Create an array of 5 equally spaced points in the interval $[0, 1]$:

```
x = linspace(0, 1, 5)
```

9. Add 1 to all 5 points:

```
1+x
```

10. Multiply all 5 points by 3:

```
3*x
```

11. Square all 5 points (Note: Octave uses a dot (.) in front of some binary operators with arrays):

```
x .^ 2
```

12. Take the square root of all 5 points:

```
sqrt(x)
```

13. Get a random number between 0 and 1:

```
rand
```

14. Get 5 random numbers between 0 and 1:

```
rand(5,1)
```

15. Store 5 random numbers between 0 and 1 in an array, determine which are < 0.5 (Note: 1 = true, 0 = false):

```
x = rand(5,1)
x < 0.5
```

16. Get the count of how many of those random numbers are < 0.5:

```
sum(x < 0.5)
```

17. Define and plot the standard normal distribution curve $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ (note the use of a semicolon at the end of the second command to suppress that command's output):

```
f = @(x) exp(-0.5*(x.^2))/sqrt(2*pi)
x = linspace(-4, 4, 5000);
plot(x, exp(-0.5*(x.^2))/sqrt(2*pi))
```

Print the plot by clicking the **Copy the plot to clipboard** icon in the upper left corner of the plot window, then paste it into either Microsoft Word or Paint and print the plot from there.

18. Integrate the standard normal distribution curve over the interval $[0,1]$ using an *adaptive Simpson's rule*:

```
quadv(f, 0, 1)
```

19. Now integrate using *Gaussian quadrature*:

```
quad(f, 0, 1)
```

20. In general, to evaluate a definite integral $\int_a^b f(x)\,dx$ in Octave, it is best to use the `quad` command, which can be used in this form:

```
quad(@(VARIABLE) FUNCTION, a, b)
```

For example, run this command to calculate $\displaystyle\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$:

```
quad(@(x) x*sin(x)/(1 + cos(x)^2), 0, pi)
```

21. Find the area of the region bounded by $y = \cos x$, $y = x$, and the y-axis. One way is to find where $\cos x = x$ (i.e. where $\cos x - x = 0$) then integrate the function $\cos x - x$ from $x = 0$ to that number. You can use the `fsolve` command to solve the equation $\cos x - x = 0$, then use that answer in the integration:

```
a = fsolve(@(x) cos(x) - x, 1)
quad(@(x) cos(x) - x, 0, a)
```

22. Another way to find the area of the same region is to use Monte Carlo integration, enclosing the region in the rectangle $[0,1]\times[0,1]$ and then selecting 10 million random points in that rectangle:

```
N = 10000000
x = rand(N,1);
y = rand(N,1);
area = 1.0*sum(y < cos(x) & y > x)/N
```

23. Enter these commands and print out the plots they create:

```
sombrero
peaks
```

24. Copy and paste your complete Octave session into a text document and print that document out.

25. Type `quit` to exit Octave.

26. Open Maxima. A window will appear where you can start entering commands. Commands in Maxima always end with a semicolon. Use Shift+Enter to submit the command (i.e. hold down the Shift key when hitting the Enter key).

The command in Maxima for taking the derivative of a function is:

```
diff(FUNCTION, VARIABLE);
```

For example, calculate the derivative of e^{-x^2} like this:

```
diff(exp(-x^2), x);
```

27. Run the commands to find the derivative of each of these functions:

(a) $e^{-2x^3/5}$ **(b)** $\ln(\sin 2x)$ **(c)** $(x^2-4x-9)^{50}$

28. To take the n^{th} derivative of a function, the command is:

```
diff(FUNCTION, VARIABLE, n);
```

Run the commands to find the second, third, and tenth derivatives of $(x^2-4x-9)^{50}$.

29. To evaluate an indefinite integral in Maxima, the command is:

```
integrate(FUNCTION, VARIABLE);
```

Run the commands to find the indefinite integrals of the following functions:

(a) $\dfrac{1}{1+e^x}$ **(b)** $\csc x$ **(c)** $\dfrac{1}{\sin x + \cos x}$ **(d)** $\ln x$ **(e)** $\sin x^2$

30. The Maxima command for computing a limit $\lim\limits_{x\to a} f(x)$ is:

```
limit(FUNCTION, VARIABLE, a);
```

The notation for ∞ and $-\infty$ are `inf` and `minf`, respectively.

For example, calculate $\lim\limits_{x\to\infty} \dfrac{x}{2x+1}$ like this:

```
limit(x/(2*x + 1), x, inf);
```

31. Go to the File menu, select Print, and print out your complete Maxima session.

32. Type `quit();` to exit Maxima.