Calculus II **Lab Assignment**

For this lab you will Octave—a free clone of the commercial numerical computing software MATLAB®—and Maxima, a free computer algebra system (CAS). You can get the latest versions of those applications from their homepages:

- Octave: `https://www.gnu.org/software/octave/`
- Maxima: `https://maxima.sourceforge.io/`

Download the appropriate versions for your operating system and follow the installation instructions. If you are using Windows then make sure when installing Octave that the **Octave Forge** component is selected for installation (if that option is available in the **Choose Components** dialog window).

Octave uses the same syntax for operations and functions as Maxima. One difference is that Octave uses the special notation `pi` for the number π (whereas Maxima uses `%pi`). Commands in Octave are submitted by the Enter key. In Octave you can use the up and down arrow keys to navigate through your command history.

Below is the syntax that Octave and Maxima use for the most common mathematical operations and functions:

Operation/Function	**Symbol/Command**	**Normal math**	**Octave syntax**	**Maxima syntax**
Addition	`+`	$2+3$	`2+3`	`2+3`
Subtraction	`-`	$2-3$	`2-3`	`2-3`
Multiplication	`*`	$2x$	`2*x`	`2*x`
Division	`/`	$\frac{x}{4}$	`x/4`	`x/4`
Exponentiation	`^`	x^3	`x^3`	`x^3`
$\sqrt{x}$	`sqrt(x)`	$\sqrt{x-1}$	`sqrt(x-1)`	`sqrt(x-1)`
e^x	`exp(x)`	e^{2x}	`exp(2*x)`	`exp(2*x)`
$\ln x$	`log(x)`	$\ln 3x$	`log(3*x)`	`log(3*x)`
$\sin x$	`sin(x)`	$\sin \pi x$	`sin(pi*x)`	`sin(%pi*x)`
$\cos x$	`cos(x)`	$\cos 4x^2$	`cos(4*x^2)`	`cos(4*x^2)`
$\tan x$	`tan(x)`	$\tan^2 x$	`tan(x)^2`	`tan(x)^2`
$\sec x$	`sec(x)`	$\sec \pi x$	`sec(pi*x)`	`sec(%pi*x)`
$\csc x$	`csc(x)`	$\csc 4x^2$	`csc(4*x^2)`	`csc(4*x^2)`
$\cot x$	`cot(x)`	$\cot^2 x$	`cot(x)^2`	`cot(x)^2`
$\sin^{-1} x$	`asin(x)`	$\sin^{-1} \pi x$	`asin(pi*x)`	`asin(%pi*x)`
$\cos^{-1} x$	`acos(x)`	$\cos^{-1} 4x^2$	`acos(4*x^2)`	`acos(4*x^2)`
$\tan^{-1} x$	`atan(x)`	$\tan^{-1} 5x$	`atan(5*x)`	`atan(5*x)`
$\sinh x$	`sinh(x)`	$\sinh \pi x$	`sinh(pi*x)`	`sinh(%pi*x)`
$\cosh x$	`cosh(x)`	$\cosh 4x^2$	`cosh(4*x^2)`	`cosh(4*x^2)`
$\tanh x$	`tanh(x)`	$\tanh^2 x$	`tanh(x)^2`	`tanh(x)^2`
$\sinh^{-1} x$	`asinh(x)`	$\sinh^{-1} \pi x$	`asinh(pi*x)`	`asinh(%pi*x)`
$\cosh^{-1} x$	`acosh(x)`	$\cosh^{-1} 4x^2$	`acosh(4*x^2)`	`acosh(4*x^2)`
$\tanh^{-1} x$	`atanh(x)`	$\tanh^{-1} 5x$	`atanh(5*x)`	`atanh(5*x)`

For this lab, follow the steps below:

1. Open Octave and enable long decimal support and disable paging of results with these commands in the Octave window:

```
format long
more off
```

Then evaluate $1 - \dfrac{\sin\left(e^{\pi^2}\right)}{\ln\left(\sqrt{4\cosh(1.2)} + \tan^{-1}(2)\right)}$ with this command:

```
1 - sin(exp(pi^2))/log(sqrt(4*cosh(1.2)) + atan(2))
```

You should see output like this:

```
ans =  0.758693256412168
```

2. Create an array of 5 equally spaced points in the interval $[0, 1]$:

```
x = linspace(0, 1, 5)
```

3. Add 1 to all 5 points (Note: Use a dot (.) in front of binary operators with arrays):

```
x .+ 1
```

4. Multiply all 5 points by 3:

```
x .* 3
```

5. Square all 5 points:

```
x .^ 2
```

6. Take the square root of all 5 points:

```
sqrt(x)
```

7. Get a random number between 0 and 1:

```
rand
```

8. Get 5 random numbers between 0 and 1:

```
rand(5,1)
```

9. Store 5 random numbers between 0 and 1 in an array, determine which are < 0.5 (Note: 1 = true, 0 = false):

```
x = rand(5,1)
x < 0.5
```

10. Get the count of how many of those random numbers are < 0.5:

```
sum(x < 0.5)
```

11. Define and plot the standard normal distribution curve $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ (note the use of a semicolon at the end of the second command to suppress that command's output):

```
f = @(x) exp(-0.5*(x.^2))/sqrt(2*pi)
x = linspace(-4, 4, 5000);
plot(x, exp(-0.5*(x.^2))/sqrt(2*pi))
```

Print the plot by clicking the **Copy the plot to clipboard** icon in the upper left corner of the plot window, then paste it into either Microsoft Word or Paint and print the plot from there.

12. Integrate the standard normal distribution curve over the interval $[0, 1]$ using the trapezoid rule with 4 trapezoids (5 points):

```
x = linspace(0, 1, 5)
y = exp(-0.5*(x.^2))/sqrt(2*pi)
trapz(x, y)
```

You should see this result:

```
ans =  0.340081844545560
```

13. Repeat with 100 trapezoids:

```
x = linspace(0, 1, 101);
y = exp(-0.5*(x.^2))/sqrt(2*pi);
trapz(x, y)
```

14. Enter the commands to repeat the above with 5000 trapezoids then with 10000 trapezoids.

15. Now integrate using an adaptive Simpson's rule:

```
quadv(f, 0, 1)
```

16. Now integrate using *Gaussian quadrature*:

```
quad(f, 0, 1)
```

17. In general, to evaluate a definite integral $\int_a^b f(x)\,dx$ in Octave, it is best to use the `quad` command, which can be used in this form:

```
quad(@(VARIABLE) FUNCTION, a, b)
```

For example, integrate $\sin^2(x^2)$ over $[0, \sqrt{100\pi}]$:

```
quad(@(x) (sin(x.^2)).^2, 0, sqrt(100*pi))
```

18. Find the area of the region bounded by $y = \cos x$, $y = x$, and the y-axis. One way is to find where $\cos x = x$ (i.e. where $\cos x - x = 0$) then integrate the function $\cos x - x$ from $x = 0$ to that number. You can use the `fsolve` command to solve the equation $\cos x - x = 0$, then use that answer in the integration:

```
a = fsolve(@(x) cos(x) - x, 1)
quad(@(x) cos(x) .- x, 0, a)
```

19. Another way is to use *Monte Carlo integration*: The region is contained inside the rectangle $[0,1] \times [0,1]$, so pick $N = 1$ million random points (x, y) in that rectangle and get the count C of how many fall inside the region. The proportion of C to N will be approximately the same as the proportion of the region's area to the rectangle's area. So the area of the region will be approximately (rectangle's area)×(C/N).

```
x = rand(1000000,1);
y = rand(1000000,1);
area = 1.0*sum(y < cos(x) & y > x)/1000000
```

Note: & = "and", | = "or"

20. Try this again with $N = 10$ million random points.

21. The area of the unit circle is $\pi(1^2) = \pi$. Use Monte Carlo integration with 1 million points to estimate that area. Bonus points for a one-liner!

22. Compute the full arc length of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$:

```
pkg load specfun
```

Matlab compatibility note: the above command is needed only in Octave, not in Matlab.

```
a2 = 9
b2 = 4
major = max(a2,b2)
minor = min(a2,b2)
k = sqrt(major)
ecc = 1.0 - minor/major
[K,E] = ellipke(ecc)
length = 4*k*E
```

23. Copy and paste your complete Octave session into a text document and print that document out. Close Octave.

24. Open Maxima. A window will appear where you can start entering commands. Commands in Maxima always end with a semicolon. Use Shift+Enter to submit the command (i.e. hold down the Shift key when hitting the Enter key).

To evaluate an indefinite integral in Maxima, the command is:

```
integrate(FUNCTION, VARIABLE);
```

For example, enter this command to evaluate $\int x\,e^{2x}\,dx$:

```
integrate(x*exp(2*x), x);
```

25. Run the commands to evaluate the indefinite integrals of the following functions:

(a) $\sinh 2x$ **(b)** $x\cos 3x$ **(c)** $\sqrt{4+x^2}$ **(d)** $\sqrt{4-x^2}$ **(e)** $\sqrt{x^2-4}$

26. The Maxima command for finding the sum of an infinite series $\sum_{n=c}^{\infty} a_n$ is:

sum(a_n, n, c, inf), simpsum;

For example, enter this command to find the sum of $\sum_{n=1}^{\infty} \frac{1}{2^n}$:

```
sum(1/2^n, n, 1, inf), simpsum;
```

27. Run the commands to find the sum of the given infinite series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$ **(c)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6}$

28. The Maxima command for finding the Taylor's series for $f(x)$ about $x=a$ up to order n (i.e. the $O(x^n)$ approximation) is:

```
taylor(FUNCTION, VARIABLE, a, n);
```

For example, enter this command to find the Taylor's series for $f(x)=\sin x$ about $x=0$ up to the 7th power:

```
taylor(sin(x), x, 0, 7);
```

29. Run the commands to find the Taylor's series for the given function $f(x)$ about the given point $x=a$ up to the order n:

(a) $f(x)=\tan x$; $a=0$; $n=9$ **(b)** $f(x)=\sec x$; $a=0$; $n=8$ **(c)** $f(x)=\sqrt{x}$; $a=1$; $n=5$

30. Go to the File menu, select Print, and print out your complete Maxima session.